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LUMINESCENCE CHARACTERISTICS OF CYLINDRICAL AND SPHERICAL LIGHT-SCATTERING MEDIA

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Abstract—Approximate analytical solutions are obtained for the equations of radiative transfer in spherical and infinite cylindrical light-scattering media with uniformly distributed radiation sources. Use is made of the source function preliminarily found to Eddington's approximation and of a number of mathematical simplifications, whose error is estimated by direct numerical calculations. The expressions for emissivities of the cylindrical and spherical media are analyzed depending on scattering properties of the medium and experimental conditions.

NOMENCLATURE

I, $= I(\tau, \theta, \varphi)$, radiation intensity at point τ and in the direction $\mathbf{l} = \mathbf{l}(\theta, \varphi)$;

B, $= B_\nu(T) = \dfrac{2h\nu^3}{c^2}(e^{h\nu/kT}-1)^{-1}$, Planck radiation intensity for frequency ν and temperature T;

J, $= J(\tau) = \dfrac{1}{4\pi}\int_{(4\pi)} I(\tau, \theta, \varphi)\,d\Omega$, mean radiation intensity;

S, $= S(\tau)$, radiation source function;

$\varepsilon_o^F(\tau_o)$(or $\varepsilon_o(\tau_o, \mu)$), total (or directional) emissivity of a plane layer; $\mu = \cos\theta$;

$\varepsilon^F(\tau_o)$(or $\varepsilon(\tau_o, \mu)$), total (or directional) emissivity of an infinite cylindrical medium;

$e^F(\tau_o)$(or $e(\tau_o, \mu)$), total (or directional) emissivity of a spherical medium;

κ and σ, absorption and scattering indices, respectively;

$\alpha = \kappa + \sigma$, attenuation index of a medium;

$\lambda = \dfrac{\sigma}{\kappa+\sigma}$, probability of quantum survival (or the Schuster number);

$0 \leqslant r \leqslant R$, cylinder or sphere radius;

$0 \leqslant \tau + \alpha r \leqslant \tau_o = \alpha R$, optical thickness of a cylinder or sphere along the radius;

$\mathbf{n}$, external normal to the boundary surface;

$I_n(x)$, Bessel functions of the nth order imaginary argument.

1. INTRODUCTION

RADIATIVE heat transfer plays an important role in studying physical characteristics of different power plants (high-temperature chemical reactors, metallurgical furnaces, boilers, etc.). The necessity to solve problems on radiative heat transfer arises also in atmospheric optics concerned with re-entry of space vehicles where radiation may not be neglected. At present operating temperature levels of power plants are increasing, that gives rise to specific requirements to precise theoretical and experimental methods for radiative heat transfer, especially those concerned with radiative and temperature fields inside the media considered since physical and chemical processes may greatly depend on radiation and temperature distribution in a medium. The improvement of the methods is primarily concerned with the study both of physical (in particular, optical) constants of the media used in power plants and of thermophysical and optical characteristics of the boundary surfaces. On the other hand, it is necessary to search for new methods and improve the available ones used for calculating radiative energy transfer. If radiation propagation in a medium is accompanied by multiple scattering processes, then great mathematical difficulties appear when solving radiative heat-transfer problems. Numerous studies show [1–3] that in some cases scattering processes play a very essential role for studying radiative properties of furnace media. It is natural since in modern power plants usually a "gas–solid particle" system serves as a heat transfer agent. More often the simplest model (that of a plane layer) is used for radiative properties of light-scattering media. The real geometry of furnace media is however more close to the axisymmetric model. Some problems of radiative energy transfer in spherically symmetric media are solved by the Monte Carlo method [4–6] or by their reduction to integral equations with subsequent numerical calculation [7–9].

Problems on luminescence of light-scattering media may be of independent interest, e.g. for spectroscopy of flames, thermal atmospheric regime, radiative fields around space apparatuses, regimes in combustion chambers of engines, etc. More often these problems are a part of more complex ones on radiative gas dynamics. So the study of radiative characteristics of supersonic two-phase flows sometimes requires simultaneous solution of gas dynamic and integrodifferential equations for radiative transfer [4]. In thermal engineering problems simultaneous consideration should be made of radiative, convective and conductive heat transfer [1–3]. Since such problems are very complicated, the necessity arises to develop correct and convenient methods for determining emissivities of light-scattering non-plane media.

Radiative characteristics of light-scattering media of cylindrical and spherical geometries with uniformly distributed radiation sources are found and analyzed here. The method proposed is based on solving the radiative transfer equation involving the source function, whose approximate form is preliminarily found. For convenience of their use and physical illustration the final results are simplified by approximate relations, whose accuracy is confirmed by direct numerical calculations.

2. SOURCE FUNCTIONS FOR CYLINDER AND SPHERE TO EDDINGTON'S APPROXIMATION

The authors [11] showed that radiative characteristics of a plane light-scattering layer of a finite optical thickness may be calculated with great accuracy in terms of the source function preliminarily calculated to Schwarzschield–Schuster's approximation. To determine emissivities of cylindrical and spherical light-scattering media, first of all, the expressions for the source functions should be found to Eddington's approximation [12–14].

Let a cylindrical or spherical medium with a radius R be characterized by some attenuation index $\alpha = \kappa + \sigma$. In the subsequent calculations, the indicatrix of radiation scattering is assumed to be spherical on the volume element of the medium under consideration. To account for non-sphericity of the scattering indicatrix in multiple processes of scattering, it is possible to introduce the following scattering function [11]:

$$p(\mu, \mu') = a + 2(1-a)\delta(\mu - \mu'), \qquad (1)$$

which reduces the initial equation to the radiative transfer one with a spherical indicatrix of scattering but with a new value of the scattering index $\sigma' = a\sigma$.

The equations for radiative transfer in infinitely cylindrical and spherical light-scattering homogeneous media are of the form, respectively [15]:

$$\sin\theta\cos\varphi \frac{\partial I(\tau,\theta,\varphi)}{\partial\tau} - \frac{\sin\theta\sin\varphi}{\tau}\frac{I(\tau,\theta,\varphi)}{\partial\varphi} + I(\tau,\theta,\varphi) = S(\tau), \qquad (2)$$

$$\cos\theta\frac{\partial I(\tau,\theta)}{\partial\tau} - \frac{\sin\theta}{\tau}\frac{\partial I(\tau,\theta)}{\partial\theta} + I(\tau,\theta) = S(\tau), \qquad (3)$$

where

$$S(\tau) = \frac{\lambda}{4\pi}\int_0^{2\pi} d\varphi \int_0^{\pi} I(\tau,\theta,\varphi)\sin\theta\, d\theta + S_o(\tau) \qquad (4)$$

is the source function due to scattering processes and intrinsic radiation of the medium. Figure 1 shows a coordinate system in the cases considered; provided local thermodynamic equilibrium (LTE), the function $S_o(\tau)$ is defined by:

$$S_o(\tau) = (1-\lambda)B. \qquad (5)$$

With no outside radiation onto the medium under consideration, the boundary condition for equations (2) and (3) is given by:

$$I(\tau_o, \mathbf{l})_{(\mathbf{ln})<0} = 0. \qquad (6)$$

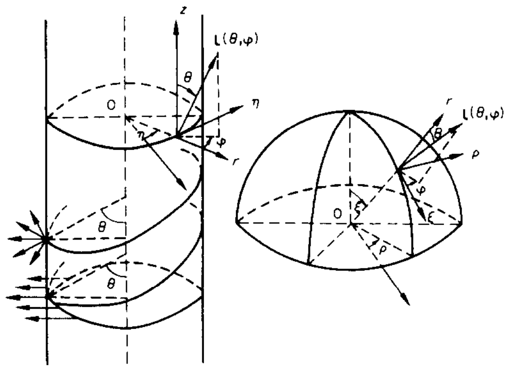

FIG. 1. Choice of a coordinate system.

As is shown in [12–14] provided LTE, the equation for radiative transfer in infinite cylindrical and spherical media may be reduced by Eddington's approximation to the following one for mean radiation intensity:

$$\Delta J(\tau) = k^2[J(\tau) - B]. \qquad (7)$$

Here

$$\Delta^s = \frac{\partial^2}{\partial\tau^2} + \frac{1}{\tau}\frac{\partial}{\partial\tau} = \frac{1}{\tau}\frac{\partial}{\partial\tau}\left(\tau\frac{\partial}{\partial\tau}\right) \qquad (8)$$

for an infinite cylindrical medium and

$$\Delta^s = \frac{\partial^2}{\partial\tau^2} + \frac{2}{\tau}\frac{\partial}{\partial\tau} = \frac{1}{\tau^2}\frac{\partial}{\partial\tau}\left(\tau^2\frac{\partial}{\partial\tau}\right) \qquad (9)$$

for a spherically symmetric medium,

$$k^2 = 3(1-\lambda). \qquad (10)$$

The boundary condition for equation (7) to Eddington's approximation may be written as:

$$\left.\frac{dJ(\tau)}{d\tau}\right|_{\tau=\tau_o} = -\tfrac{3}{2}J(\tau_o). \qquad (11)$$

The solution of problems (7)–(11) lead to the following relations for a mean intensity of radiation propagating in cylindrical and spherical light-scattering media, respectively:

$$J_c(\tau) = B[1 - AI_o(k\tau)], \quad J_s(\tau) = B\left(1 - C\frac{\tau_o \sinh k\tau}{\tau\sinh k\tau_o}\right), \qquad (12)$$

where

$$A^{-1} = I_o(k\tau_o) + \tfrac{2}{3}I_1(k\tau_o), \quad C^{-1} = \frac{1}{\tau_o}\left(\tau_o - \tfrac{2}{3} + \tfrac{2}{3}k\tau_o\frac{1+e^{-2k\tau_o}}{1-e^{-2k\tau_o}}\right). \qquad (13)$$

Hence, the unknown source functions are found for light-scattering cylinder and sphere, respectively:

$$S_c(\tau) = \lambda J_c(\tau) + (1-\lambda)B = B[1 - \lambda AI_o(k\tau)], \qquad (14)$$

$$S_s(\tau) = B\left(1 - \lambda C\frac{\tau_o\sinh k\tau}{\tau\sinh k\tau_o}\right). \qquad (15)$$

Substitution of equations (14) and (15) into equations (2) and (3), respectively, and variables of the type

$v = \tau\cos\varphi$ and $w = \tau\sin\varphi$ result in the following solution of the problem stated:

$$\frac{1}{B}J_c(\tau,\theta,\varphi) = 1-\exp\left(-\frac{\sqrt{[\tau_o^2-\tau^2\sin^2\varphi]}+\tau\cos\varphi}{\sin\theta}\right) -\lambda A\exp\left(-\frac{\tau\cos\varphi}{\sin\theta}\right) \times\int_{-\sqrt{(\tau_o^2-\tau^2\sin^2\varphi)}}^{\tau\cos\varphi} I_o(k\sqrt{[x^2+\tau^2\sin^2\varphi]}) \times\exp\left(\frac{x}{\sin\theta}\right)\frac{\mathrm{d}x}{\sin\theta}, \quad (16)$$

$$\frac{1}{B}J_s(\tau,\mu) = 1-\exp(-\tau\mu-\sqrt{[\tau_o^2-\tau^2+\tau^2\mu]})-\frac{\lambda C\tau_o}{\sinh k\tau_o} \times\int_{-\sqrt{(\tau_o^2-\tau^2+\tau^2\mu^2)}}^{\tau\mu}\frac{\sinh(k\sqrt{[x^2+\tau^2-\tau^2\mu^2]})}{\sqrt{(x^2+\tau^2-\tau^2\mu^2)}} \times\exp(x-\tau\mu)\,\mathrm{d}x. \quad (17)$$

Note that the expression for the radiation intensity in a cylindrical medium at $\sin\theta = 0$ (radiation propagates along the cylinder axis) may be obtained directly from equation (2): $I_c = S_c(\tau)$.

3. EMISSIVITY OF INFINITE CYLINDRICAL LIGHT-SCATTERING MEDIUM

From expression (16) emissivity of a cylindrical light-scattering medium may be easily found

$$\varepsilon = \varepsilon(\tau_o,\theta,\varphi) = \frac{1}{B}I_c(\tau_o,\theta,\varphi) = 1-\mathrm{e}^{-2a}-\lambda a A_1\Psi, \quad (18)$$

where

$$a = \frac{\tau_o\cos\varphi}{\sin\theta}, \quad A_1^{-1} = 1-\tfrac{2}{3}k\frac{I_1(k\tau_o)}{I_o(k\tau_o)},$$
$$\Psi = \int_{-1}^{1}\frac{I_o(k\tau_o\sqrt{[x^2\cos^2\varphi+\sin^2\varphi]})}{I_o(k\tau_o)}\mathrm{e}^{ax}\,\mathrm{d}x. \quad (19)$$

Relation (18) at $\lambda = 1$ (pure scattering medium) and at $\lambda = 0$ (no light-scattering) is consistent with the physical meaning of the problem studied. However, it is very difficult to study luminescence characteristics of a cylindrical medium by the above relation. Such investigation becomes even more complicated due to the necessity to integrate expression (18) with respect to the angle φ since in many cases of practical importance the distance from a cylinder to the receiver is much greater than its diameter. Note that on physical grounds integration with respect to φ from $-\pi/2$ to $\pi/2$ answers summation of cylinder radiation in a certain direction (Fig. 1). For integration of intensity of emerging radiation with respect to φ use is made of the approximate equality which is strictly valid for propagation of diffusional radiation [15] in the plane light-scattering layer

$$\int_0^1 J(\tau_o,\mu)\,\mathrm{d}\mu \cong J(\tau_o,\tfrac{1}{2}). \quad (20)$$

This equality also holds for emissivity of a plane light-scattering layer of a finite optical thickness [11]. The validity of this condition is also confirmed by direct numerical integration of the Ψ-function with respect to φ. Therefore, for the Ψ-function integrated with respect to φ, it may be written:

$$\int_0^1 \Psi(\tau_o,\theta,\mu)\,\mathrm{d}\mu \cong = \int_{-1}^{1}\frac{I_o(k\tau_o\cdot\frac{1}{2}\sqrt{[3+x^2]})}{I_o(k\tau_o)}\mathrm{e}^{ax}\,\mathrm{d}x. \quad (21)$$

To obtain the final expression for emissivity of a cylindrical medium, use is made of the following approximate relations:

$$\frac{I_o(\alpha\,.\,k\tau_o)}{I_o(k\tau_o)} \cong \mathrm{e}^{-\frac{1}{2}(1-\alpha^2)k\tau_o} \quad \text{for} \quad 0{\cdot}85 \leqslant \alpha \leqslant 1,$$
$$\frac{I_1(x)}{I_o(x)} \cong 1-\mathrm{e}^{-x/2}, \quad (22)$$

$$\int_{-1}^{1}\exp[-a(1-x)(1+bx)]\,\mathrm{d}x \cong \frac{1}{a}\left(1-\frac{2ab}{3a+2b}\right)(1-\mathrm{e}^{-2a}) \text{ for } 0 \leqslant b \leqslant 0{\cdot}3. \quad (23)$$

Direct calculations of relations (22) and (23) verify their validity. So, Table 1 contains calculation results on relation (22). The error of relations (22) is seen to be $\lesssim$ 10 per cent. As the error signs are different, the above error may be diminished when calculating the final expressions.

Table 1. Calculation from approximate relations (16) (ex. and app. are exact and approximate functions)

	$I_1(x)/I_o(x)$		$I_o(\alpha x)/I_o(x)$					
x			$\alpha = 0{\cdot}85$		$\alpha = 0{\cdot}90$		$\alpha = 0{\cdot}95$	
	ex.	app.	ex.	app.	ex.	app.	ex.	app.
0·5	0·243	0·221	0·98	0·93	0·99	0·95	0·99	0·98
1·0	0·447	0·394	0·94	0·87	0·96	0·91	0·98	0·95
1·5	0·595	0·528	0·88	0·81	0·92	0·87	0·96	0·93
2·0	0·695	0·632	0·82	0·76	0·87	0·83	0·93	0·91
2·5	0·765	0·714	0·76	0·71	0·83	0·79	0·91	0·88
3·0	0·810	0·777	0·70	0·66	0·79	0·75	0·89	0·87
4·0	0·861	0·865	0·60	0·57	0·71	0·68	0·84	0·82
6·0	0·911	0·950	0·44	0·44	0·58	0·57	0·76	0·75
8·0	0·934	0·982	0·33	0·33	0·47	0·47	0·69	0·68
10·0	0·950	0·993	0·24	0·25	0·39	0·39	0·62	0·61

Thus, for the emissivity of an infinite cylindrical light-scattering medium we have

$$\varepsilon = \varepsilon(\tau_o, \theta) = 1 - e^{-\frac{\tau_o}{\sin\theta}} - \frac{\left(1 - \frac{\frac{1}{2}k\tau_o\delta}{3k\tau_o + \delta^2}\right)\left(1 - e^{-\frac{k\tau_o}{\delta}}\right)}{\left(1 + \frac{k}{4}\sin\theta\right)\left[1 + \frac{2}{3}k\left(1 - e^{-\frac{k}{2}\tau_o}\right)\right]}, \quad (24)$$

where

$$\delta = \frac{k\sin\theta}{1 + \frac{k}{4}\sin\theta}, \quad k = \sqrt{[3(1-\lambda)]}. \quad (25)$$

Figures 2 and 3 show angular distribution of radiation intensity emitted by an infinite cylindrical light-scattering medium at different values of probability of quantum survival λ and optical radius $\tau_o = (\kappa + \sigma)R$. For comparison the emissivity of a plane layer with the similar optical properties but at $\tau_o' = 2\tau_o$ is presented in these figures, from which it is seen that the strongest difference in the nature of angular distribution is observed for weakly scattering media of small optical thicknesses. The emissivity of a light-scattering cylinder increases with the optical thickness slower than in the case of a plane layer (Fig. 4). Attention should be paid to the upper part of Figure 4 where are presented the calculations from relation (18) (solid curves) at $\sin\theta = \frac{1}{2}$ and emissivities of a plane layer (dotted curves) [11]:

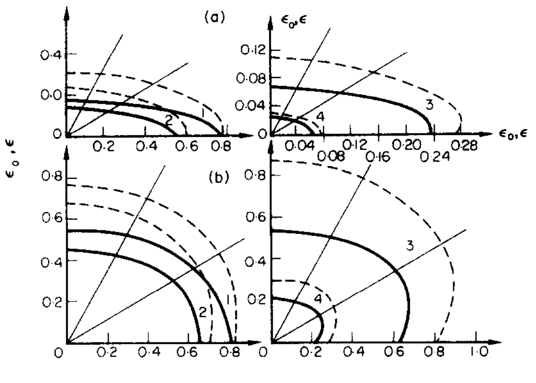

FIG. 2. Angular distribution of emissivities of cylindrical ε (solid curves) and plane ε_o (dotted curves) light-scattered media a, $\tau_o = 0{\cdot}5$; b, $\tau_o = 2{\cdot}0$; 1, $\lambda = 0{\cdot}3$; 2, $\lambda = 0{\cdot}5$; 3, $\lambda = 0{\cdot}8$; 4, $\lambda = 0{\cdot}95$.

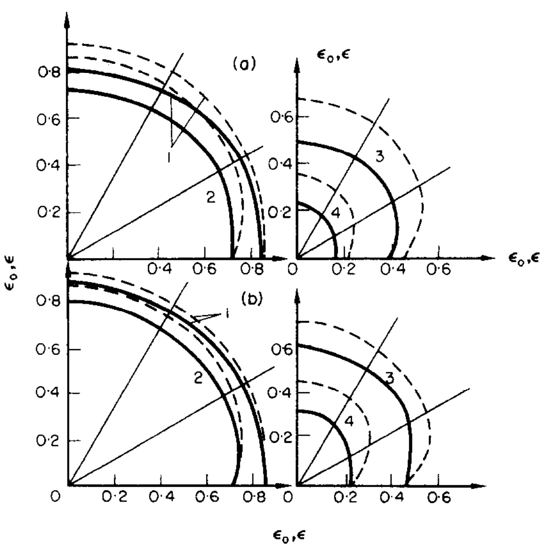

FIG. 3. Angular distribution of emissivities of cylindrical ε (solid curves) and plane ε_o (dotted curves) light-scattering media a, $\tau_o = 5$; b, $\tau_o = 15$.

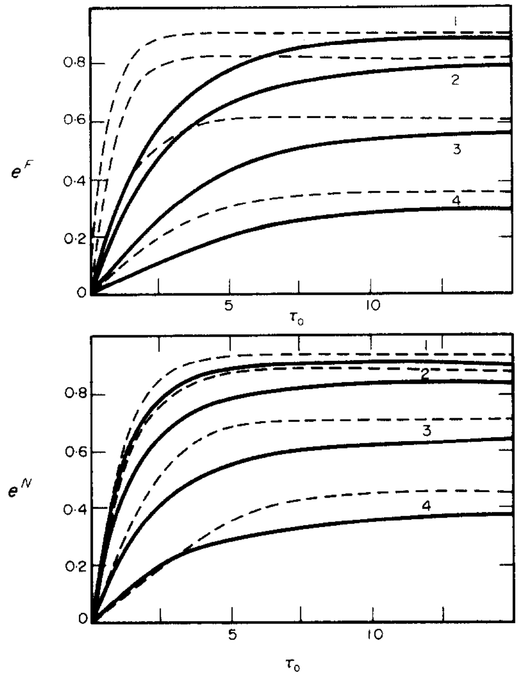

FIG. 4. Plot of total (ε^F) and normal (ε^N) emissivities of a cylindrical medium vs optical thickness; dashed line, emissivity of a plane layer ($\tau_o' = 2\tau_o$) 1, $\lambda = 0{\cdot}3$; 2, $\lambda = 0{\cdot}5$; 3, $\lambda = 0{\cdot}8$; 4, $\lambda = 0{\cdot}95$.

$$\varepsilon_o(\tau_o, \theta) = 1 - e^{-\frac{\tau_o}{\sin\theta}} - \frac{\lambda}{1 + k_o + (1-k_o)e^{-2k_o\tau_o}} \times \left[\frac{1 - e^{-\frac{\tau_o}{\sin\theta} - 2k_o\tau_o}}{1 + 2k_o\sin\theta} + \frac{e^{-2k_o\tau_o} - e^{-\frac{\tau_o}{\sin\theta}}}{1 - 2k_o\sin\theta}\right],$$

$$k_o = \sqrt{(1-\lambda)}. \quad (26)$$

The values of the total emissivity of a plane layer [11]:

$$\varepsilon_o^F(\tau_o) = (1-R)\frac{1 - e^{-2k_o\tau_o}}{1 + R e^{-2k_o\tau_o}}, \quad R = \frac{1-k_o}{1+k_o} \quad (27)$$

are denoted by circles in Fig. 4. Good agreement between the quantities $\varepsilon_o^F(\tau_o)$ and $\varepsilon_o(\tau_o, \pi/6)$ confirms the reliability of the accepted condition for intensity averaging over the angle for radiating light-scattering media. This agreement also implies that the data on a light-scattering cylinder at $\theta = \pi/6$ depicted in Fig. 4 may be taken as its total emissivity given by:

$$\varepsilon^F(\tau_o) = 1 - e^{-2\tau_o} - \frac{8\lambda\left[1 - \frac{2k\tau_o(8+k)}{16k + 3\tau_o(8+k)^2}\right]\left[1 - e^{-\frac{\tau_o}{4}(8+k)}\right]}{3(8+k)^2\left[3 + 2k\left(1 - e^{-\frac{k}{2}\tau_o}\right)\right]}. \quad (28)$$

In case of infinitely thick optical medium, equation (26) is substituted by

$$\varepsilon(\tau_o, \theta)\bigg|_{\tau_o \to \infty} = 1 - \frac{4\lambda(12 + k\sin\theta)}{(3+2k)(4+k\sin\theta)^2}. \quad (29)$$

Table 2 shows that the calculations from the above expression agree with emissivities of a semi-infinite layer calculated with a spherical scattering indicatrix on a volume element of a substance, i.e. at $a \equiv 1$ [11]:

$$\varepsilon_o(\tau_o, \theta)\bigg|_{\tau_o \to \infty} = \frac{1 + 2\sin\theta}{1 + 2\sqrt{(1-\lambda)}\sin\theta}\sqrt{(1-\lambda)}. \quad (30)$$

Table 2 contains the values of total emissivities of a semi-infinite layer and a cylinder

$$\varepsilon_o^F(\tau_o)\bigg|_{\tau_o \to \infty} = \frac{2\sqrt{(1-\lambda)}}{1+\sqrt{(1-\lambda)}}$$

and (31)

$$\varepsilon^F(\tau_o)\bigg|_{\tau_o \to \infty} = 1 - \frac{8\lambda(24+k)}{(3-2k)(8+k)^2}.$$

It should be noted that it is more correct to compare emissivities of a layer ε_o and a sphere e due to physical essence of the choice of the coordinate system (Fig. 1).

With the known values of $\varepsilon(\tau_o, \theta)$ and $\varepsilon^F(\tau_o)$ it is not difficult to determine directional and total intensity of radiation emitted by a light-scattering cylinder

$$I_c(\tau_o, \theta) = \varepsilon(\tau_o, \theta)\pi RB, \quad I_c(\tau_o) = \tfrac{1}{2}\varepsilon^F(\tau_o)\pi RB. \quad (32)$$

To study the effect of experimental conditions on the value of luminescence intensity of a light-scattering cylinder, expression (18) is numerically calculated depending on angles O and C, i.e. with different directions of observation. Moreover, the calculation made is also important to check relations (24) and (28) which determine directional and total emissivities of a cylindrical medium. The effect of experimental conditions on the emissivity of a light-scattering cylinder is shown in Fig. 5 where the increase in the angle φ considerably changes the emissivity of a cylinder at $\theta \sim 90°$, i.e. when radiation is observed in the directions close to the normal to the lateral cylinder surface. As should be expected, at $\theta \to 0°$ (or $\theta \to 180°$) emissivity does not depend any more on the angle φ. As the optical radius increases, the emissivity tends to a certain limit, the value and rate of approaching the limit being determined both by optical characteristics of the medium considered and by experimental conditions, i.e. by the angles θ and φ.

Note that with no radiation ($\lambda = 0$) the values of total emissivity of a cylindrical medium coincide with those of work [2] within 7 per cent over the whole range of the optical thickness.

Table 2. Comparison of calculated emissivities of plane ε_o, cylindrical ε and spherical e light-scattering media of infinitely large optical thickness

	$\sin\theta = 1$			$\sin\theta = 0{\cdot}8$			$\sin\theta = 0{\cdot}5$			$\sin\theta = 0$		
λ	ε_o	ε	e	ε_o	ε	e	$\varepsilon_o = \varepsilon_o^F$	$\varepsilon = \varepsilon^F$	$e = e^F$	ε_o	ε	e
0·1	0·982	0·973	0·982	0·980	0·970	0·979	0·974	0·965	0·974	0·949	0·952	0·952
0·2	0·962	0·942	0·961	0·957	0·937	0·956	0·945	0·926	0·945	0·894	0·902	0·902
0·3	0·939	0·908	0·938	0·930	0·899	0·929	0·911	0·884	0·911	0·837	0·847	0·847
0·4	0·912	0·868	0·910	0·899	0·857	0·898	0·873	0·836	0·874	0·775	0·789	0·789
0·5	0·879	0·822	0·876	0·862	0·808	0·861	0·828	0·782	0·829	0·707	0·725	0·725
0·6	0·838	0·767	0·834	0·817	0·750	0·815	0·775	0·720	0·776	0·632	0·653	0·653
0·7	0·784	0·698	0·780	0·759	0·678	0·756	0·708	0·644	0·709	0·548	0·571	0·571
0·8	0·708	0·606	0·703	0·678	0·584	0·674	0·618	0·547	0·620	0·447	0·472	0·472
0·9	0·581	0·467	0·574	0·546	0·445	0·542	0·480	0·409	0·482	0·366	0·341	0·341
0·95	0·463	0·352	0·456	0·528	0·333	0·424	0·365	0·302	0·367	0·223	0·245	0·245
0·99	0·250	0·173	0·243	0·223	0·161	0·220	0·182	0·144	0·183	0·100	0·112	0·112

4. ANALYSIS OF LUMINESCENCE CHARACTERISTICS OF LIGHT-SCATTERING SPHERE

According to (17) directional emissivity of a light-scattering sphere is equal to:

$$e(\tau_o, \mu) = \frac{1}{B} I_s(\tau_o, \mu) = 1 - e^{-2\tau_o\mu} - \lambda CF(\tau_o, \mu) \quad (33)$$

where

$$F(\tau_o, \mu) = \frac{\tau_o}{\sinh k\tau_o} \int_{-\tau_o\mu}^{\tau_o\mu} \frac{\sinh(k\sqrt{[x^2 + \tau_o^2 - \tau_o\mu^2]})}{x^2 + \tau_o^2 - \tau_o\mu^2} \times e^{-(\tau_o\mu - x)}\,dx, \quad (34)$$

$$\mu = \cos\theta', \quad \theta' = \frac{\pi}{2} - \theta.$$

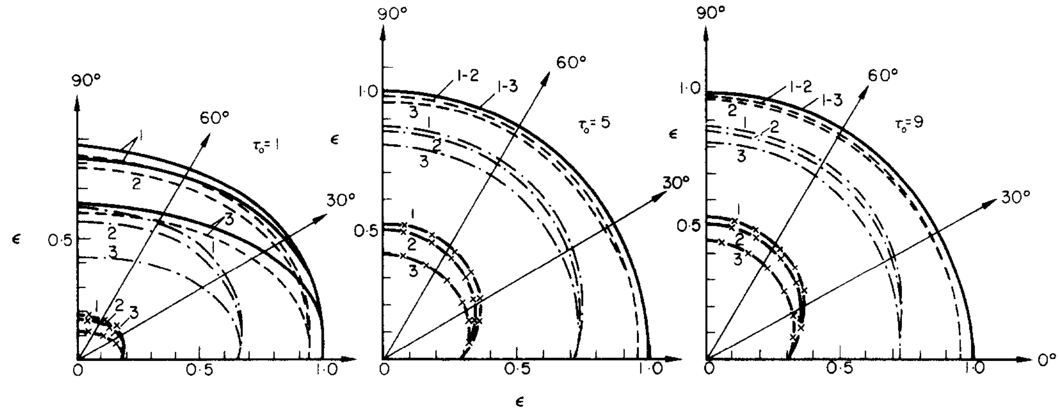

FIG. 5. Angular distribution of radiation of a light-scattering cylindrical medium at different angles φ 1, $\varphi = 0°$; 2, $\varphi = 30°$; 3, $\varphi = 60°$; ——, $\lambda = 0$; – – –, $\lambda = 0{\cdot}1$; –·–·– $= 0{\cdot}5$; —×—×— $= 0{\cdot}9$.

Since determination of total emissivity of a light-scattering sphere requires integration over the angle, more convenient representation of function (34) should be found. So, see Table 3.

$$F(\tau_o, \mu) \cong \frac{1 - e^{-2\tau_o \mu}}{1 + \dfrac{\mu k \tau_o}{3k + \tau_o}}. \qquad (35)$$

appears to be the simplest representation.

With regard for expression (35) the emissivity of a light-scattering sphere in some direction is given by:

$$e(\tau_o, \mu) = (1 - e^{-2\tau_o \mu})\left(1 - \frac{\lambda C}{1 + \dfrac{\mu k \tau_o}{3k + \tau_o}}\right) \qquad (36)$$

The functions

$$e(\mu) = e(\tau_o, \mu)\Big|_{\tau_o \to \infty} = 1 - \frac{\lambda}{(1 + \frac{2}{3}k)(1 + \mu k)} \qquad (37)$$

and emissivities of a semi-infinite layer from (20) presented in Table 2 show that the extreme case $\tau_o \to \infty$ of relation (36) is physically valid.

Use of expression (36) and proposed condition of averaging diffusively propagating radiation over the angle (equation (20)) yields total and normal emissivities of a light-scattering medium

$$e^N = e(\tau_o, \mu)\Big|_{\mu = 1} = (1 - e^{-2\tau_o})\left[1 - \frac{\lambda C(3k + \tau_o)}{3k + (1 + k)\tau_o}\right], \qquad (38)$$

$$e^F(\tau_o) = (1 - e^{-\tau_o})\left[1 - \frac{2\lambda C(3k + \tau_o)}{6k + (2 + k)\tau_o}\right]. \qquad (39)$$

Figure 6 shows a plot of normal and total emissivities of a spherical light-scattering medium versus optical radius $\tau_o = (\kappa + \sigma)R$ as well as values of emissivity of a plane layer of optical thickness $\tau_o' = 2\tau_o$ (dotted curves).

Table 3. Values of functions (34) (ex.) and (35) (app.)

		0·1		0·3		0·5		0·7		0·9	
τ_o	θ'/λ	ex.	app.	ex.	app.	ex.	app.	ex.	app.	ex.	app.
	15°	1·75	1·70	1·75	1·70	1·75	1·70	1·76	1·71	1·76	1·71
	30°	1·59	1·55	1·59	1·55	1·59	1·55	1·59	1·55	1·59	1·55
0·1	45°	1·32	1·29	1·32	1·29	1·32	1·29	1·32	1·29	1·32	1·29
	60°	1·95	1·94	1·95	1·94	1·95	1·94	1·95	1·94	1·95	1·94
	75°	0·50	0·50	0·50	0·50	0·50	0·50	0·50	0·50	0·50	0·50
	15°	0·68	0·68	0·71	0·68	0·75	0·68	0·79	0·69	0·83	0·71
	30°	0·68	0·66	0·71	0·67	0·74	0·67	0·77	0·68	0·80	0·70
1·0	45°	0·67	0·63	0·68	0·64	0·70	0·64	0·72	0·64	0·74	0·66
	60°	0·59	0·56	0·60	0·56	0·61	0·56	0·62	0·56	0·63	0·57
	75°	0·40	0·38	0·40	0·38	0·40	0·38	0·40	0·38	0·40	0·38
	15°	0·27	0·34	0·30	0·34	0·33	0·35	0·38	0·36	0·45	0·38
	30°	0·30	0·34	0·32	0·35	0·35	0·35	0·39	0·36	0·45	0·38
2·0	45°	0·33	0·35	0·35	0·36	0·38	0·36	0·41	0·37	0·45	0·39
	60°	0·36	0·35	0·37	0·35	0·39	0·36	0·40	0·36	0·42	0·38
	75°	0·31	0·29	0·31	0·29	0·31	0·29	0·32	0·29	0·32	0·30
	15°	0·040	0·048	0·044	0·051	0·048	0·054	0·055	0·058	0·070	0·069
	30°	0·043	0·051	0·047	0·053	0·051	0·056	0·058	0·061	0·073	0·071
10·0	45°	0·049	0·056	0·052	0·058	0·057	0·061	0·064	0·066	0·077	0·075
	60°	0·059	0·065	0·063	0·066	0·067	0·069	0·073	0·073	0·084	0·081
	75°	0·078	0·077	0·081	0·079	0·083	0·081	0·087	0·083	0·093	0·089

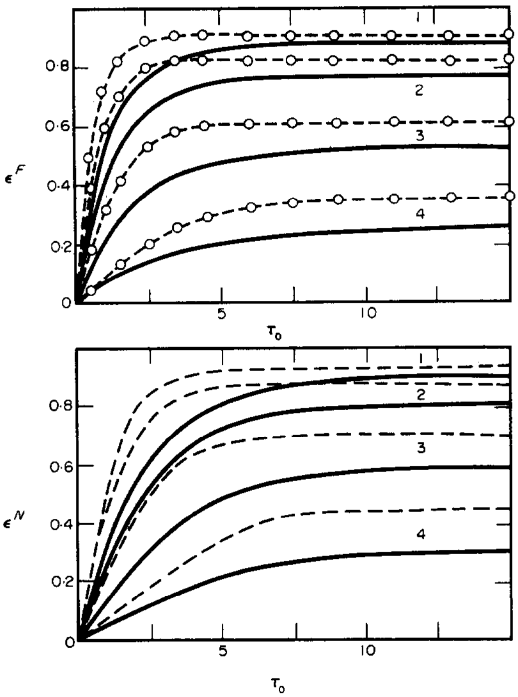

FIG. 6. Plot of normal (e^N) and total (e^F) emissivities of a spherical medium vs optical radius $\tau_o = (\kappa+\sigma)R$; dashed line, emissivity of a plane layer ($\tau_o' = 2\tau_o$) 1, $\lambda = 0{\cdot}3$; 2, $\lambda = 0{\cdot}5$; 3, $\lambda = 0{\cdot}8$; 4, $\lambda = 0{\cdot}95$.

For calculation of radiation intensity of a light-scattering sphere use should be made of the following relations:

$$I(\tau_o, \mu) = e(\tau_o, \mu)B, \quad I(\tau_o) = \tfrac{4}{3}\pi R^2 e^F(\tau_o)B. \quad (40)$$

Note that with no regard for scattering, according to equations (26), (28) and (39), relations of total emissivities of a layer, cylinder and sphere may be written as:

$$\varepsilon_o^F(\tau_o'): \varepsilon^F(\tau_o): e^F(\tau_o) = (1-e^{-4\tau_o}): (1-e^{-2\tau_o}): (1-e^{-\tau_o})$$

where $\tau_o' = 2\tau_o = 2\kappa R$. The calculations from this relation agree satisfactorily with the available reported values [2].

5. CONCLUSION

The relations proposed for emissivities of cylindrical and spherical light-scattering media are very simple and may therefore be used to solve problems on radiative gas dynamics and combined heat transfer. The method used in this paper which is essentially an iteration procedure based on preliminary determination of the source function when used for solving the equation of radiative transfer with uniformly distributed radiation sources for a plane layer is very accurate. Due to the assumptions accepted the error of the method increases for a light-scattering cylinder and sphere. It is difficult to estimate the value of this error because of no necessary data. The physical correctness of the results obtained is confirmed by considering the extreme cases (pure radiating media, comparison of the results with regard for light scattering with large optical thicknesses, etc.). The errors of the method may approximately be estimated by comparing the values obtained with the exact ones. The analysis made shows that for a cylindrical medium the errors of calculation of emissivity in the most unfavourable situations is about 20 per cent. For spherical media the error of the method may be estimated by comparing expressions (34) and (35), see Table 3. On the other hand, as is seen from the dependences of emissivities of light-scattering cylinder and sphere upon optical properties of a medium and experimental conditions, use of the plane layer approximation for real light-scattering cylindrical or spherical objects may result in the error which grows with an increase of the light-scattering contribution in the media under investigation.

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CARACTERISTIQUES DE LA LUMINESCENCE DES MILIEUX DIFFUSIFS CYLINDRIQUES ET SPHERIQUES

Résumé—Des solutions analytiques approchées sont obtenues pour les équations du transfert par rayonnement dans les milieux diffusifs sphériques et cylindriques infinis avec des sources de rayonnement unifomément réparties. On utilise la fonction source introduite en premier lieu dans l'approximation de Eddington et un certain nombre de simplifications mathématiques dont l'erreur qu'elles entrainent est estimée directement par le calcul numérique. Les expressions du pouvoir émissif des milieux cylindriques et sphériques sont analysées en fonction des propriétés diffusives du milieu et des conditions expérimentales.

STRAHLUNGSVERHALTEN ZYLINDRISCHER UND SPHÄRISCHER LICHTSTREUENDER MEDIEN

Zusammenfassung—Es werden analytische Näherungslösungen für die Wärmeübertragung durch Strahlung in sphärischen und durch unendlich lange Zylinder gebildeten lichtstreuenden Medien mit gleichmäßig verteilten Strahlungsquellen angegeben. Es wird dabei von der (vor der Näherungslösung von Eddington gefundenen) Quellen-Funktion und von einigen mathematischen Vereinfachungen Gebrauch gemacht, deren Fehler mittels direkter numerischer Rechnung abgeschätzt wird. Die Ausdrücke für Emissionskoeffizienten der sphärischen und zylindrischen Medien werden bezüglich der Streueigenschaften des Mediums und der experimentellen Bedingungen untersucht.

ХАРАКТЕРИСТИКИ СВЕЧЕНИЯ ЦИЛИНДРИЧЕСКИХ И СФЕРИЧЕСКИХ СВЕТОРАССЕИВАЮЩИХ СРЕД

Аннотация — Для уравнений переноса излучения в сферической и бесконечной цилиндрической светорассеивающих средах с равномерно распределенными источниками излучения получены приближенные аналитические решения. При их получении использована функция источников, предварительно найденная в приближении Эддингтона, а также ряд математических упрощений, погрешность которых оценена непосредственными численными расчетами. Выражения для излучательных способностей цилиндрических и сферических сред проанализированы в зависимости от рассеивающих свойств среды и условий проведения экспериментов.